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THE TRUE ATOMIC WEIGHT OF BROMINE.

(PLATES XXXIV.-XXXVI.)

BY DR. GUSTAVUS HINRICHES.

(*Read April 4, 1913.*)

Highly important laboratory work, undertaken for the purpose of determining the atomic weight of bromine, has quite recently been done by Dr. H. C. P. Weber.¹ Ten complete syntheses of hydrogen bromide were made, taking from 60 to 80 grammes of bromine for the individual determinations.

Employing the method of reduction in general use by the dominant school, Mr. Weber finds the atomic weight of bromine to be 79.924 with the insignificant "*probable error*" of 0.0014, oxygen at 16 exactly being taken as the standard.² Accordingly, the atomic weight of bromine should fall between 79.923 and 79.925, for O = 16.

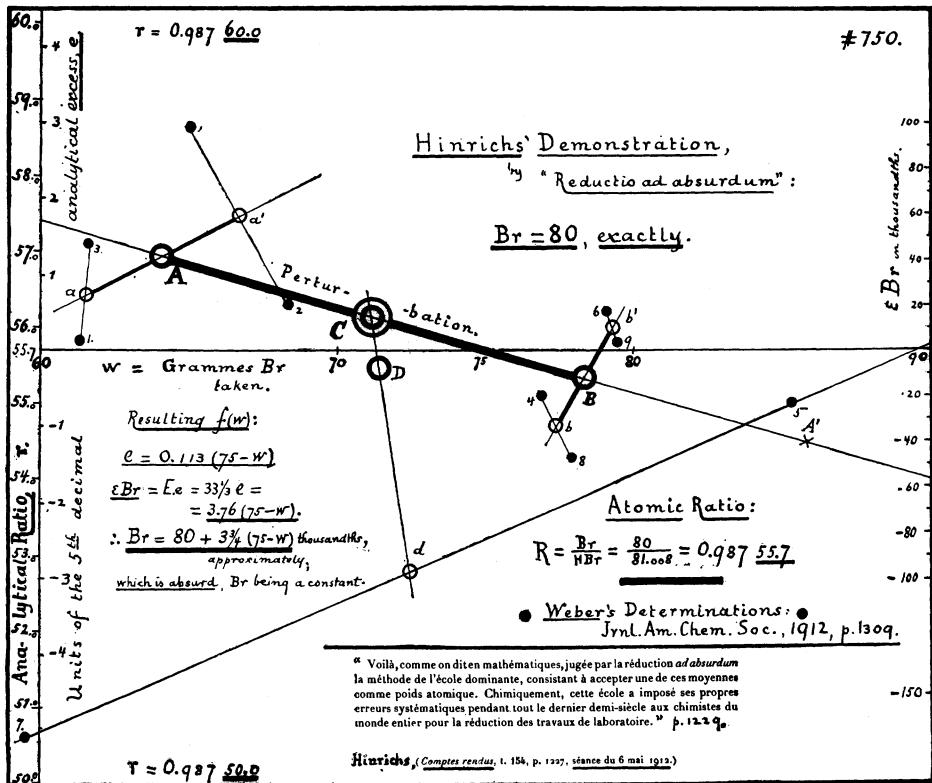
Now bromine is one of the ten fundamental elements of the system of Stas (Ag, Pb, Na, Ka; Cl, Br, Io; N, S, O). If the above value for Br should be found to be in error, such error would affect the values of most of the other nine elements also.

During the last twenty years, I believe to have demonstrated,³ by close mathematical examination of all the atomic weight determinations made during the entire century (since Berzelius began this work in 1810) that the method in common use for the *reduction* of

¹ *Jrnl. Am. Chem. Soc.*, Oct., 1912, pp. 1294-1310.

² *L. c.*, pp. 1309-1310.

³ *Special Works*: "True Atomic Weights," 1894; "Absolute Atomic Weights," 1901; "Proximate Constituents," 1904. *Twenty-five Notes* in the *Comptes Rendus*, in twelve years from 1892 to 1912; (in nine years no note on atomic weights); *Moniteur Scientifique*, thirteen papers from 1906-1909; *Revue générale de Chimie*, 1910, on hydrogen; *Proceedings American Philosophical Society*, 1910, 1911; *Proceedings Am. Assoc. Adv. Science*, 1869.



the laboratory work done is not correct but false in principle and erroneous in its results.

Very naturally the dominant school has first denounced my work and thereafter ignored the same; nevertheless it has been compelled to admit the existence of grave errors in the results of Stas which had been extolled to be of astronomical precision. This applies especially to the most famous of the fundamental determinations of Stas, namely those of Ag and N. For nitrogen, Stas gave the value 14.044 exactly. By a marvelous series of decimals (from 38 to 375 places furnished him by A. Quetelet) he declared the lowest *possible* value to be 14.040; at present, the school of Stas has come down to 14.008 which is one fifth of the lowest possible value of Stas and only 8 thousandths above the value we believe to have proved to be the true value, namely 14 *exactly*.⁴ For silver the value of Stas has been reduced by his school from 107.930 to 107.880, which is a reduction of fifty thousandths. It must be borne in mind that this matter is a question of high precision, questioning the thousandths of the unit of atomic weights.

All the above values refer to the oxygen standard in common use, O = 16 exactly, for which we believe to have proved that Ag is 108 *exactly* and Br 80 *exactly*. Hence the present values of the dominant school would be 0.120 low for Ag, *i. e.*, 0.11 per cent. of 108; and 0.076 low for Br, *i. e.*, 0.10 per cent. of 80.

If our results are correct, the dominant school is *one tenth of one per cent. low* on the atomic weight of these two fundamental elements.

Accordingly, if our work be true, all the quantitative chemical analyses made in the chemical laboratories throughout the world, from the lowest technical to the highest scientific institutions, have for half a century been falsified (unintentionally, of course, but *de facto*) to the extent of *one tenth of one per cent.* for both silver and bromine determinations. For lithium, the error committed is now fully one per cent.

⁴ The experiments of Guye and his students at Geneva are claimed to prove N = 14.008; but each set of determinations has been made within very narrow limits and with small weights at that, except those of 1912, which positively prove N = 14.000, as I have shown (*Comptes Rendus*, May 6, 1912; T. 154, p. 1227).

The question here raised is therefore of the highest practical as well as scientific importance; most assuredly, it cannot be settled by a ballot, though such has recently been taken. Nor should it be left to the decision of a select few for each country, but every individual chemist should, on this as on any other important chemical question, try to study sufficiently to enable him to form an opinion of his own.

To facilitate such a study on the part of the individual chemist, we here present the laboratory work of Mr. Weber on bromine in the most simple and direct way possible, without any refined technicalities: *simply plotting the experimental results themselves* (the ratios of the actual weights) *according to the weight of the bromine taken and the ratios found in each case.*

When the individual chemist inspects this diagram of the actual weights taken and the ratios found, he will realize that we have neither hypotheses to make nor theories to defend. We simply have tried to look at the actual experimental data obtained in the laboratory itself. We have divided the entire process into twelve consecutive steps, each one being distinct and preparatory to the next.

I. THE WEIGHTS, TAKEN AND FOUND.

TABLE I.
WEIGHTS, IN GRAMMES, TO ONE-TENTH MGR.

No.	H	Br	HBr	Discrepancy, ⁵ Mgr.
1	0.7730	61.2884	62.0605	-0.85
2	0.8606	68.2503	69.1114	0.54
3	0.7761	61.5573	62.3220	-1.42
4	0.9693	76.8822	77.8514	-0.15
5	1.0755	85.2956	86.3709	-0.15
6	0.9969	79.0683	80.0642	-0.99
7	0.7497	59.4528	60.2050	2.59
8	0.9816	77.8555	78.8376	0.43
9	1.0013	79.3963	80.3966	-0.06
10	0.8198	65.0214	65.8387	-2.56
Sum	9.0037	714.0572	723.0583	-2.63
Means	0.9004	71.4057	72.3058	-0.26

⁵ The discrepancy is: HBr—(H + Br) and its theoretical value is 0, of course. The actual value is once over 1 and twice over 2 mgr.: hence the hundredth mgr. of weighings have properly been dropped by me in copying the weights given by Weber.

II. THE ANALYTICAL RATIO.

TABLE II.

THE ANALYTICAL RATIO, r ; CALCULATED TO 6 DECIMALS, THE SIXTH AS FIRST TO THE FIFTH, SINCE ORDINARILY ONLY FIFTH IS TAKEN.

	Ratio:	1st		2d		3d	
	Br (to 1st dec.)	H/Br		H/HBr		Br/HBr	
1	61.3	0.012	61.2	0.012	45.6	0.987	55.8
2	68.3		60.9		45.2		56.3
3	61.4		61.0		45.3		57.1
4	76.9		60.7		45.1		55.1
5	85.3		60.9		45.2		55.0
6	79.1		60.8		45.1		56.2
7	79.5		61.0		45.2		50.6
8	77.9		60.8		45.1		54.3
9	79.4		61.1		45.4		55.8
10	65.0		60.8		45.2		58.6
Mean	71.4		60.9		45.2		55.5
Range	23.8		0.4		0.5		8.0

Remarks.—(1) Ratios very concordant; range small, especially in first and second (five significant digits only), less so in third (six significant digits). (2) The reciprocal ratios of nos. 1 and 2 would magnify the minute error in H eighty-fold. (3) Complete synthesis gives the three equally important ratios; Weber omits no. 3.

III. THE VARIATION OF THE ANALYTICAL RATIO.

As soon as the individual determinations of the analytical ratio r (Table II.) are plotted according to a convenient scale it is seen that this ratio is not constant, but variable. In our drawing (No. 750) the abscissæ represent the weight w of bromine taken on the scale of a centimeter to the gramme, while the ordinates represent the corresponding ratio r on the scale of one inch to the unit of the fifth decimal; that is, the unit-ratio itself is 100,000 inches which is 8,333 feet or 1.58 english mile. Of the three distinct diagrams we shall here insert only the one representing the third ratio, Br/HBr, which is the sharpest and therefore the most decisive. The reduction to centimeter scale by photography is here inserted; for this cut the unit ratio is therefore one kilometer, and the unit of the fifth decimal one centimeter. The gramme is represented by nearly four millimeters. See Plate XXXIV.

Each full black circle represents the determination identified by the same numeral used by Mr. Weber.

The single determinations for less than 60 (no. 7) and for more than 80 grammes (no. 5) of bromine give the mean d of comparatively little importance.

The eight determinations made with between 60 and 80 grammes of bromine fall in two well-defined groups of four determinations each and give the equally important two mean values marked A (1, 3; 2, 10) and B (4, 8; 6, 9) of which C is the final mean.⁶

But it is clear that this mean C cannot be considered to be the true mean value of the ratio r because it is nothing more than the mean of eighth determinations which Mr. Weber made with the eighth weights of bromine which he "happened to take." For as a matter of fact, the eight determinations do not give some constant value from which the individual determinations differ by small deviations equally distributed as to amount and sign. On the contrary, the eight determinations form a well-defined straight line $A-B$, inclined to the axis. Accordingly, it is *this straight line A-B* itself which represents the eight determinations made by Mr. Weber. Only in case the line $A-B$ were parallel to the axis of weights taken (horizontal in the drawing) and if, at the same time, all deviations were small, could the point C be taken as a legitimate mean.

It is readily seen that the line AB is the geometrical representation of the equation

$$r = 57.7 - 0.113w, \quad (1)$$

where r is expressed in units of the fifth place of decimals and w in grammes of bromine taken. That is: r is *not* a constant, but varies according to the form $k - cw$, where k and c are constants,

We may express the strange fact revealed in the above by saying that the result of the experiment depends on the choice of the weights taken by the chemist, so much so that "we can tell as soon as the weight has been taken and before the experiment has been

⁶ It may be noted that of the two isolated determinations 7 and 5, the first (7) is made with the smallest weight of bromine and therefore shows the greatest deviation, while the last (5) made with the greatest weight of bromine falls distinctly close to the line AB .

made, what the outcome thereof will be," or perhaps more strikingly still, we might say that "we get (within sufficiently large limits to count) whatever value r we would like to get."

But the value of r directly determines the value of the atomic weight itself, as we shall show in detail; hence the fact just stated for the ratio applies with equal force to the atomic weight itself.

IV. SYSTEMATIC ERRORS AND CHEMICAL PERTURBATIONS.

The existence of such systematic errors in the most refined laboratory work of renowned chemists, from Stas to the present, is not a new discovery, for I have proved the existence thereof twenty years ago. See the note presented by Berthelot at the Séance of the twelfth of December, 1892.⁷

I here insert (Plate XXXVI.) a reduction to half the original scale of the diagram (no. 215) published in the note just mentioned, together with the diagram (no. 216) of the next note (February 27, 1893). The new cut (no. 752) is a like reduction of Plate I. of my "True Atomic Weights" of 1894 and represents the systematic errors of Stas in his famous syntheses of silver nitrate (no. 251) and of lead nitrate (no. 252). See plate XXXV.

Indeed it is even forty years since I first pointed out the existence of definite *perturbations* (or disturbances) in the chemical work of Stas, namely at the Salem meeting of the American Association for the Advancement of Science in 1869,⁸ of which the part here in question is reprinted in my "True Atomic Weights," 1894, pp. 65–69, under the regretfully appropriate heading: *vox clamantis in deserto*.

We shall, however, in the future restrict the term "chemical perturbation" to such systematic errors as are expressible by a definite function and therefore representable by a definite curve. Such are the systematic errors in the recent work of Mr. Weber, being represented analytically by an equation of the first degree (1) and geometrically by the straight line $A-B$ (cut no. 750, Plate XXXIV.).

⁷ *Comptes Rendus*, T. 115, p. 1074.

⁸ *Proceedings*, pp. 112–124.

PLATE I.
Reduced to $\frac{1}{2}$ from:
The True Atomic Weights of the Chemical Elements.

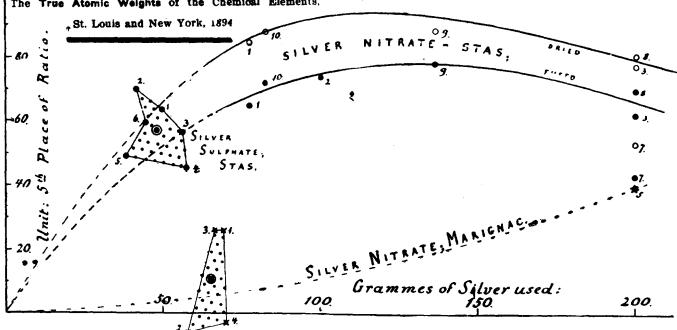
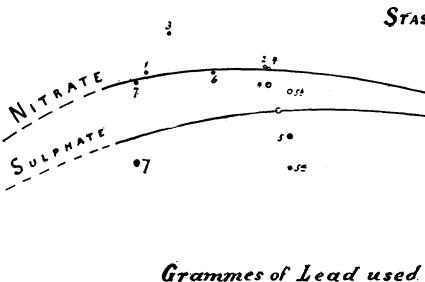


Fig. 1.
251.

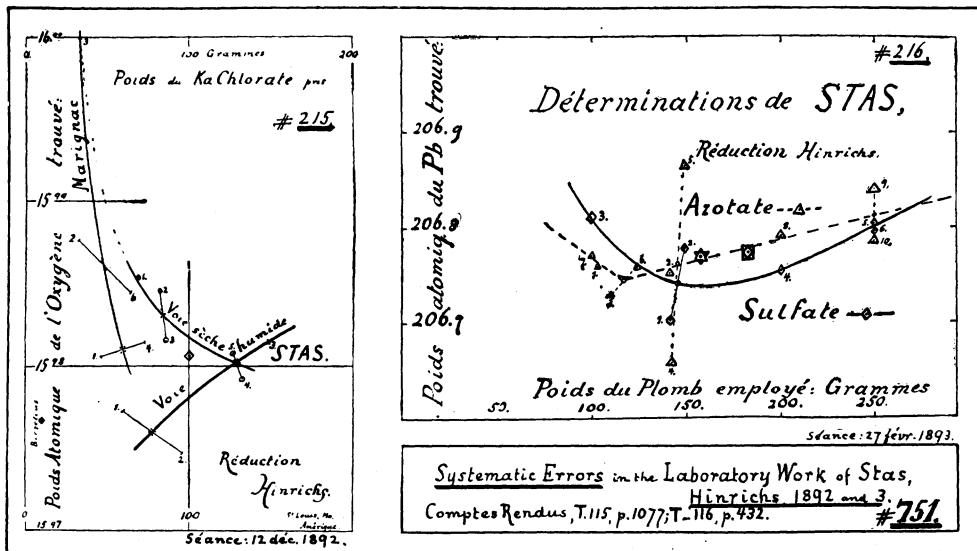
Unit: 5th Place of Ratio.



STAS' Syntheses.

This Reduction to $\frac{1}{2}$:
Hinrichs, 1912. #752

Fig. 2.
252.



In fact I have communicated to prominent chemists in the United States and in France first proofs of some of my new cuts representing quite a number of such chemical perturbations. All the dominant elements⁹ and a number of the others have now been so represented. These first proofs form already quite an atlas. One of these cuts (no. 737) has been published in the *Comptes Rendus* of the meeting of May 6, 1912.¹⁰ It covers work of Richards at Harvard and of Guye at Geneva, on the dominant elements Ag, Cl and N, O, respectively.

By this work we have also introduced into chemistry the method of demonstration of the geometricians of ancient Greece known as the "*reductio ad absurdum*," which is just as decisive in modern chemistry of precision as in geometry, the highest science of ancient Greece.

V. THE RATIONAL METHOD OF REDUCTION.

This was first published in the *Comptes Rendus* of March 27, 1893 and in my "True Atomic Weights" of 1894. It has been extended and perfected during twenty years, but no complete exposition of all the steps involved having been published in one place at one time, it is no doubt somewhat difficult to grasp and use the same. It is for this reason that we here give, *merely as an example*, its application to the laboratory work of Mr. Weber on bromine.

The old way of successive substitution, producing of necessity an accumulation of errors unknown in magnitude, is based upon the elementary method of solving an algebraic equation with supposedly one unknown only, when in fact it contains as many unknown as there are elements present in the chemical reaction employed. Since, however, in a chemical reaction it is impossible to accept any one element as without error in its action, all these equations are de facto indeterminate (or diophantic) and therefore insoluble. The solutions given by the school are therefore erroneous and cannot be in accord with the facts.

But while the work of the school during the entire century has thus necessarily failed to give a true solution of the problem by that

⁹ *Comptes Rendus*, T. 153, p. 817; 30 Oct., 1911.

¹⁰ T. 154, p. 1228.

method, it has established incidentally one general fact of great practical importance, namely that all the atomic weights are approximations to certain whole or half numbers, if the atomic weight of oxygen is taken at 16 exactly.

Consequently we may say that the entire mathematical problem will require only the exact determination of the value of this small *departure* (which we now represent by the Greek letter epsilon ϵ).

Every mathematician knows that all relations, even the most complex, are thereby reduced to simple proportions. Hence all our calculations can be carried out by proportional parts, if the necessary relations have first been deduced either by geometry or by development into series. We have used both methods. After overcoming these difficulties we have systematized the work by simple analytical processes, retaining however the general geometrical method for the presentation of the data of experiment and the results of calculation, as exemplified above and in our numerous diagrams of which reductions by photography are printed.

It seems best, at this point, to state the degree of precision aimed at: the third decimal (thousandths) of the atomic weight and the fifth decimal (hundred thousandths) of the ratios (atomic R and analytic r). If at any time we feel authorized to go beyond this general limit, the higher decimals are given as decimals to the above, in order to conform to definitions given and to avoid confusion.

VI. THE DEPARTURE, ϵ .

The true atomic weight of bromine is known to be some value quite near the number 80; all chemists admit this as an established fact. *Hence we limit our work to the determination of the precise small number of thousandths of the unit, our departure ϵ .*

Accordingly we say: the exact atomic weight of bromine is $80 + \epsilon$. We then perform all analytical operations with this sum instead of using the one symbol Br. Thus many terms will cancel and others will drop out as minute quantities of too high an order to be of influence on the result: facts and processes familiar to all those versed in mathematical work. In this way we finally obtain readily workable formulæ. (See p. 61 of our "Cinquantenaire,"

1910, for an example and note the interesting story of its wanderings in 1907 and 1908.) This may be sufficient for the present to direct those readers who may need such reference.

We now return to the simple practical details necessary for carrying out the work of determining the small departure ϵ for bromine.

VII. THE ATOMIC RATIO R , AND ITS VARIATION Δ

To express the chemical reaction used, we first take the departure as zero (that is $\text{Br}=80$ exactly and $\text{H}=1.008$ to the nearest thousandth), as we have shown it to be in our history of all determinations for hydrogen made in the century past.¹¹ This gives us R and Δ as shown in Table III. by elementary mathematics.

TABLE III.

Ratio for	1st H/Br	2nd H/HBr	3rd; Br/HBr
<i>First, for</i> Atomic ratio	$\text{Br}=80$ 1.008 $\underline{80}$	and 1.008 $\underline{81.008}$	$\text{H}=1.008$ 80 $\underline{81.008}$
which is			
$R =$	0.012 60.1	0.012 44.4	0.987 55.7
<i>Second, for</i> Atomic ratio	$\text{Br}=80.1$ 1.008 $\underline{80.1}$	and 1.008 $\underline{81.108}$	$\text{H}=1.008$ 80.1 $\underline{81.108}$
which is			
$R' =$	0.012 58.5	0.012 42.9	0.987 57.2
hence			
$\Delta \text{Br} = R' - R =$	— 1.6	— 1.5	+ 1.5
<i>Third, for</i> Atomic ratio	$\text{Br}=80$ 1.008 $\underline{80}$	and 1.008 $\underline{81.108}$	$\text{H}=1.008$ 80.0 $\underline{81.108}$
which is			
$R'' =$	0.013 85.1	0.013 66.2	0.986 33.9
hence			
$\Delta \text{H} = R'' - R$ is	125.0	121.8	— 121.8

¹¹ *Revue gén. de Chimie*, 1910, 377–389.

VIII. THE ANALYTICAL EXCESS e .

This is the number of units of the fifth decimal by which r exceeds R ; that is

$$e = r - R. \quad (2)$$

Table II. gives the following results:

TABLE IV.
VALUES OF THE ANALYTICAL EXCESS, e .

No. $R =$	W Br (grammes)	$\frac{1}{H/Br}$ 0.012 60.1	$\frac{2}{H/HBr}$ 0.012 44.4	$\frac{3}{Br/HBr}$ 0.987 55.7
1	61.3	1.1	1.2	0.1
2	68.3	0.8	0.8	0.6
3	61.4	0.9	0.9	1.4
4	76.9	0.6	0.7	-0.6
5	85.3	0.8	0.8	-0.7
6	79.1	0.7	0.7	0.5
7	59.5	0.9	0.8	-5.1
8	77.9	0.7	0.7	-1.4
9	79.4	1.0	1.0	0.1
10	65.0	0.7	0.8	3.0
Mean	71.4	0.8	0.8	-0.2

The concordance between r (experiment) and R (absolute value) is almost perfect, $e = r - R$ affecting the millionths almost exclusively (in all but four of the thirty cases!).

If we were to decide by mean values, in accordance with the common practice of the school, we would have to conclude that *the true atomic weight of bromine is 80 exactly*; for the mean analytical excess is only 8 millionths above in the first and second ratios and merely 2 millionths below in the third ratio. This implies a truly marvelous approximation to our absolute atomic weight $Br=80$ exactly.

Since the third ratio is mathematically the sharpest, being near unity, we might claim in truth having demonstrated with "astronomical precision" that the atomic weight of bromine is 80 exactly.

By means of the known value of the variation Δ we can readily convert the value of the analytical excess into the corresponding departure ϵ in thousandths of the unit of atomic weights.

IX. THE VARIATION OF THE ANALYTICAL EXCESS, e .

The mere fact that the mean value of the excess e is very small (0.2 only) is, however, not sufficient to completely establish the conclusion just drawn, as we have always accentuated; for some of the individual values of the excesses might be large with opposite sign, or show notable *systematic variations*—this they do in fact in the present case.

Hence it is necessary to study the individual values of the analytical excess for each laboratory determination made. They are given in Table IV. of section VIII., in the last column. By simply drawing (on diagram no. 750) the base line (horizontally) through the point on the scale of the analytical ratios for the value of the atomic ratio, $R = 0.98755.7$ (see VII.), we get the representation of e on the diagram. This horizontal line is shown on our cut no. 750, which thus exhibits the exact value of the analytical excess for every individual determination made. See Plate XXXIV.

Taking this line as the new axis of abscissæ for e the equation of the line of perturbation will be

$$e = 0.113(75 - w). \quad (3)$$

It is not necessary to discuss this equation, since it corresponds exactly to that for r already considered in III.

X. THE INCREMENT, Σ .

Σ is the change in the third decimal of the atomic weight produced by one unit in the fifth place of the analytical ratio r .

It will be impossible to enter here upon the complete deduction of the formulæ now to be presented for practical work; for our deduction we must refer to a series of papers in the *Comptes Rendus* from 1907 to the present and to our publications in other quarters, especially including the facsimile of a general deduction which has travelled between Paris and St. Louis in 1907 and again in 1908; see p. 61 of my “Cinquantenaire de L’atomécanique, 1910,” under the title: *Un Manuscrit Voyageur*. All these more recent develop-

ments of general formulæ really go back to 1894 in my "The Atomic Weights," pp. 157-161. On p. 159 will be found the formula (42) for the chemical perturbation, essentially the same as the one we have been using for a number of years.

In fact, it would be interesting to trace the development we have been able to make of the method of Lagrange so renowned with mathematicians and astronomers under the name of "The Method of the Variation of the Arbitrary Constants."

Our simplest formula, obtained by means of Taylor's most general formula, for m chemical elements present in the reaction, is

$$\Sigma \epsilon \Delta = 100e. \quad (4)$$

Treating the effect of the elements ex-aquo, this equation becomes

$$m \epsilon \Delta = 100e, \quad (5)$$

or simply

$$\epsilon = \Sigma e; \quad (6)$$

if we introduce the increment Σ as defined above

$$\Sigma = 100/m\Delta, \quad (7)$$

which also may be defined as the departure per unit of the excess e .

In the case under consideration we have the variation determined in VIII., while the number of elements present (m) is 2 (Br and H); hence the third reaction (the sharpest) gives the values of the departures presented in the next section XI. as Table V.

XI. SUMMARY OF RESULTS OBTAINED FOR THE REACTION $R = \text{Br} : \text{HBr}$.

TABLE V.

I. THE ARBITRARY CONSTANTS a (i. e., THE ABSOLUTE ATOMIC WEIGHT) AND THEIR VARIATION (Δ , Σ).

	Elements	
	Br	H
Absolute atomic weight, a	80	1.008
Variation, Δ (units of fifth place)	1.5	— 121.8
Increment, Σ (units of third place)	33 $\frac{1}{3}$	— 0.41
Departure, ϵ , by e	33 $\frac{1}{3}e$	— 0.41e

II.—MEAN VALUES OF DEPARTURES, ϵ .

Point	Experiments	w	ϵ	ϵ_{Br}	ϵ_H
A	1, 2, 3, 10	64.0	1.25	41.5	- 0.51
B	4, 6, 8, 9	78.3	- 0.35	- 11.7	0.14

Mean of groups A and B.

C	A, B	71.61	0.45	15.0	- 0.18
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Mean of all Determinations.

D	C, d	71.77	- 0.22	- 7.3	0.09
	<i>Departure as function of w' = 75 - w</i>			$3.77w'$	$- 0.05w'$

III.—MEAN ATOMIC WEIGHTS.

Point	Experiments	w	ϵ	Br	H
A	1, 2, 3, 10	64.0	1.25	80.042	1.0075
B	4, 6, 8, 9	78.3	- 0.35	79.988	1.0081

General Means.

The 2 groups of four determinations each:

C	A, B	71.61	0.45	80.015	1.0078
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All ten determinations:

D	C, d	71.77	- 0.22	79.993	1.0081
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IV.—TRUE ATOMIC WEIGHT.

For all possible weights, i. e. entire line:

Point	Experiments	w	ϵ	Br	H
A — B	1,2,3,4,6,8,9,10	71.61	0.00	80.000	1.0080

Note.—Confirmatory determinations desirable especially for weights symmetrically taken in reference to A and B; that is for mean weights of about 86 and 72 grammes.

Reference: "True Atomic Weights," 1894, p. 158.

XII. GENERAL CONCLUSION.

It will have been noticed that our work really gives *three* distinct demonstrations that the atomic weight of Bromium is 80 exactly.

First (as shown above in VIII.), by the *limit-method*, the means closely approaching the value Br = 80 exactly.

This method was first published in the *Comptes Rendus* from 1892 to 1894 when it was more fully developed in my book: "The True Atomic Weights," 1894, of which Berthelot accepted the dedication.

Second. By the "Chemical Perturbations," representing all determinations by a single straight line, intersecting the axis of abscissæ of weights taken.

This also demonstrates completely that the true atomic weights *cannot* be determined by empirical methods alone.

In fact, the empirical methods lead really more and more away from the truth by the chemists trying to secure greater concordance—which can most easily be reached by limiting the range of weights taken (as already shown here) and in many other ways, all leading into error. We hope soon to take up this most important practical subject more thoroughly.

Third.—By the *reductio ad absurdum*, showing that the atomic weight for the individual determination is *de facto* a function of the weight taken for effecting the determination; this result is evidently absurd, because the atomic weight in its very nature is independent of the amount or weight of the substance operated upon.

In other words: the individual determinations establish the line of perturbation only, the intersection of which with the axis of weights taken (*for e=0*) gives the true solution for all experiments represented on that line of perturbations.

Final Conclusion. While each one of these three demonstrations, taken separately, is sufficient to prove that the true atomic weight of bromine is 80 exactly, they properly constitute three consecutive steps in *one complete demonstration* which itself has been gradually developed in that order above given. It may be best to repeat them here as links of that chain of demonstration:

1. The values of the analytical excess *e* are minute and of opposite sign, giving a mean more or less closely approaching to zero; hence the horizontal line *e=0*, determined theoretically by the atomic ratio *R*, is the locus of the true atomic weight.

2. The straight line of perturbation is a second locus of the same; *hence the intersection of these two lines determines the weight for which the laboratory work is without error.*

3. The other parts of the line of perturbation give the atomic weight as function of the weight taken for the experiment, which being absurd, proves that they only serve to determine the point

of intersection, as just done. Besides: each single point in the line of perturbation *above* the point of intersection is balanced by the equal value of contrary sign *below* that line symmetrical with the above point.

Trusting that we have made this subject as clear as so difficult a matter—at first reading—may be made, we shall add only that all thought of the so-called “probable error” of the mean must be laid aside in atomic weight determinations; we have repeatedly shown its utter absurdity in this field—the more careful laboratory work having the desperate character of placing itself squarely far beyond the field restricted for it by the so-called “probable error of the mean” so as to leave it not a shred of probability. We expect as soon as possible, by a most striking example (covering all the work done for an important element) to show the utter fallacy of this “probable error of the mean” introduced by Gauss a century ago. We shall, at the same time, show that it is but a false and misleading substitute for the simple mean value of the actual distance of each determination from the mean of all.

We here insert, from a most extended table carefully classified by order of magnitude and for each individual element separately, giving the most general results in a broader grouping, to show the fact referred to above: that with progress in practical laboratory work, the final departure has been greatly reduced.

The four catalogues of work represent: (*A*) recent and classical work (Dumas, Stas to present); (*B*) older determinations (except the classical); (*C*) dominant elements only: *i. e.*, O, Cl, Ag; C, Na; S, Br, H; N, Ka.

PER CENT. OF EACH GRADE SPECIFIED.

Grade in Words	ϵ	Catalogue				per 100 per 100
		B	AB	A	C	
Excellent to good	Below 100	77	85	93	98	
Poor to bad	Above 100	23	15	7	2	

Number of:				
Determinations.....	508	967	459	159
Reactions.....	163	340	177	53